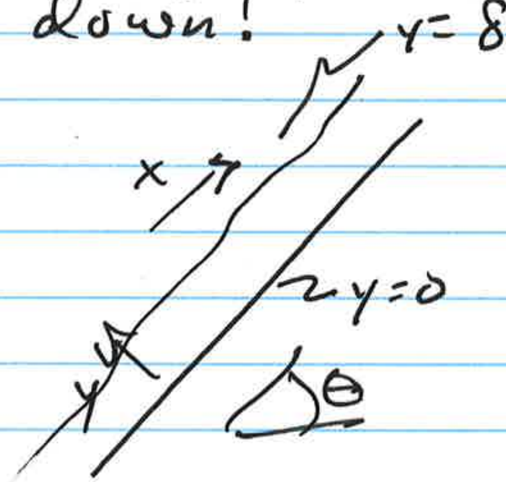
Tears of Wine Demo:

How thick is the film?

This is complex but we can get an approximate upper bound by looking at a balance between Marangoni flow going up & gravity driven flow going down!



For a thin film:

$$0 = \mu \frac{\partial^2 u_x}{\partial y^2} - \rho g \sin\theta$$

w/ BCs:

$$u_x \Big|_{y=0} = 0$$

and

$$\mu \frac{\partial u_x}{\partial y}\Big|_{y=\delta} = \tau_0 \equiv \frac{\Delta \Gamma}{H}$$

change in surf. tension

length in x-direction

Let's <u>scale</u>

$$y^* = y/\delta_c \;,\quad \delta^* = \delta/\delta_c \;,\quad u_x^* = \frac{u_x}{U_c}$$

↳ <u>scale</u> of layer thickness, δ!

So:

$$0 = \frac{\mu U_c}{\delta_c^2}\frac{\partial^2 u_x^*}{\partial y^{*2}} - \rho g \sin\theta$$

Divide:

$$0 = \frac{\partial^2 u_x^*}{\partial y^{*2}} - \left[\frac{\rho g \sin\theta\, \delta_c^2}{\mu U_c}\right]$$

$$\text{''} \atop 1$$

<u>So</u> $U_c = \dfrac{\rho g \sin\theta\, \delta_c^2}{\mu}$

Now for BCs:

$$u_x^*\Big|_{y^*=0} = 0$$

$$\frac{\mu U_c}{\delta_c}\frac{\partial u_x^*}{\partial y^*}\Big|_{y^*=\delta^*} = \frac{\Delta\Gamma}{H}$$

Divide out:

$$\left.\frac{\partial u_x^*}{\partial y^*}\right|_{y^*=\delta^*} = \frac{\Delta\Gamma}{H}\frac{\delta_c}{\mu U_c} = \underbrace{\left[\frac{\Delta\Gamma}{H\rho g\sin\theta\,\delta_c}\right]}_{=1}$$

So $\delta_c = \dfrac{\Delta\Gamma}{H\rho g\sin\theta}$!

Now we have:

$$\frac{\partial^2 u_x^*}{\partial y^{*2}} = +1 \qquad \left.u_x^*\right|_{y^*=0} = 0, \quad \left.\frac{\partial u_x^*}{\partial y^*}\right|_{y^*=\delta^*} = 1$$

Solving:

$$u_x^* = \frac{1}{2}y^{*2} + Ay^* + B$$

since $\left.u_x^*\right|_{y^*=0} = 0$ $\therefore$ $B = 0$!

and at δ^*:

$$\left.\frac{\partial u_x^*}{\partial y^*}\right|_{y^*=\delta^*} = \left.y^* + A\right|_{y^*=\delta^*} = \delta^* + A = 1$$

$$\therefore A = 1 - \delta^*$$

So:

$$u_x^* = y^* - \left(\delta^* y^* - \tfrac{1}{2} y^{*2}\right)$$

shear flow upwards

gravity driven flow downwards

What is the <u>flow rate</u>?

$$\frac{Q}{W} = \int_0^{\delta} u_x \, dy = U_c \delta_c \int_0^{\delta^*} u_x^* \, dy^*$$

So

$$\frac{Q/W}{U_c \delta_c} = \frac{1}{2}\delta^{*2} - \left(\frac{1}{2}\delta^{*3} - \frac{1}{6}\delta^{*3}\right)$$

$$= \frac{1}{2}\delta^{*2} - \frac{1}{3}\delta^{*3}$$

Note that Q/W is <u>zero</u> if $\delta^* = 0$ (no film!) <u>and</u> if $\delta^* = 3/2$

Q/W is <u>maximum</u> when $\underline{\delta^* = 1}$

So what is δ_c?

$$\delta_c = \frac{\Delta\Gamma}{H \rho g \sin\theta}$$

<u>guess</u> $\frac{\Delta\Gamma}{H} \simeq 1 \text{ dyne/cm}^2$

$\rho = 1 \text{ g/cm}^3$, $g = 980 \text{ cm/s}^2$, $\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \delta_c = 0.0014 \text{ cm} = 14 \mu m$

This is <u>about</u> the thickness if you pre-swirl the wine! If it just results from creep up from the meniscus it is less than this value — but it gives you a reasonable starting point!